Epistemic Logics with Strategies

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- Wang [2015,2018] proposed a framework for knowing how logics.

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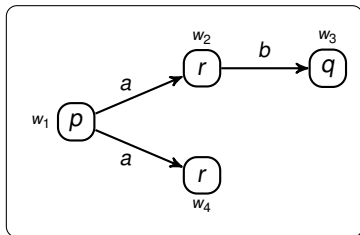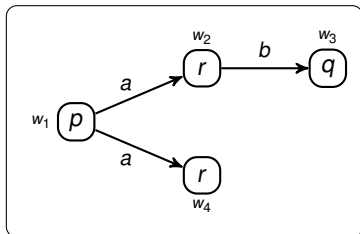
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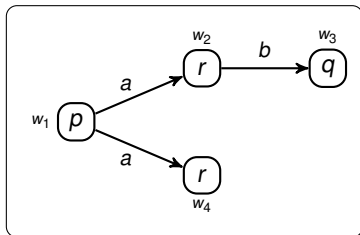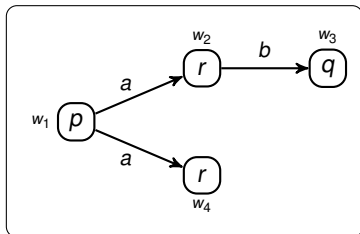
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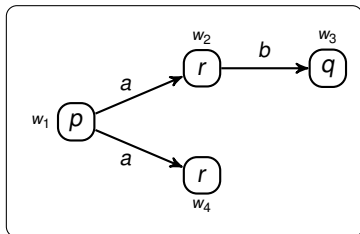   plan *a* takes the agent from every
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C. Areces, R. Fervari, A. R. Saravia, F. R. Velázquez-Quesada. *Uncertainty-Based Semantics for Multi-Agent Knowing How Logics.* (TARK 2021).

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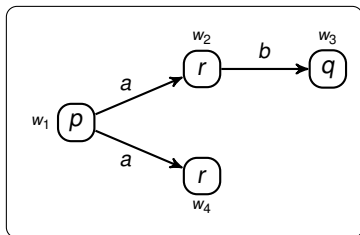$\mathcal{M} \models \text{Kh}_i(\psi, \varphi)$ iff exists a set of plans π such that

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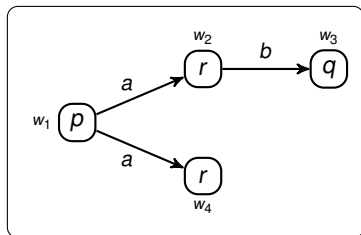- **(3)** *from ψ-states, every plan in π always ends in φ-states.*

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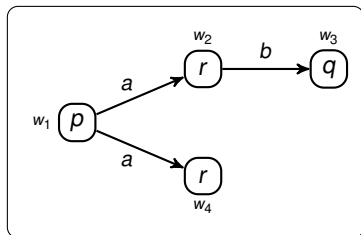- **(3)** *from ψ-states, every plan in π always ends in φ-states.*

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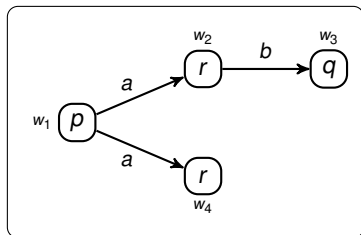
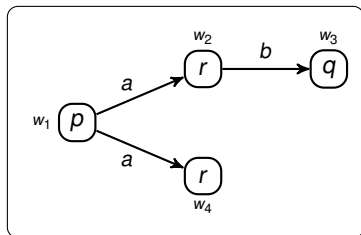$\mathcal{M} \not\models \text{Kh}_i(p, r)$ (given $a \sim_i ab$)
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∴ {a, ab} is not fail-proof at w_1

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    Moreover:
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    - SAT is NP-complete.
    - Strongly complete axiom system.
    - Weaker than the original proposal (but a more general logic).

Ongoing works

- Dynamic operators: Learning/forgetting how.
    with C. Areces, R. Fervari, F. Velázquez-Quesada (DaLí 2022).
- Reinterpretation in Deontic Logic: Knowingly complying
    with C. Areces, V. Cassano, P. Castro, R. Fervari
        (accepted for AAMAS 2023).